N71-29026

NASA TECHNICAL TRANSLATION NASA TT F- 13,612

INVESTIGATION OF ORDINARY WAVES IN A CHEW, GOLDBERGER, AND LOW APPROXIMATION

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Translation of "Issledovaniye prostykh voln v priblizhenii Chu, Gol'dbergera i Lou," Report Pr-23, Institute for Space Research, Academy of Sciences, USSR, Moscow, 1970, 22 pages

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 MAY 1971

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ABSTRACT: The Chew, Goldberger, and Low hydrodynamic equations for collisionless plasma are used to investigate ordinary waves (or Riemann waves). The integral curve field for fast and slow magneto-sonic waves is investigated in detail. It is shown that in a fast magneto-sonic wave the magnetic field increases with increase in density, just as is the case in magnetohydrodynamics with isotropic pressure. The slow magneto-sonic wave has a region in which the magnetic field drops to the compression wave, as well as an anomalous region in which the sign of the derivative changes.

It is shown that fast and slow magneto-sonic waves have a tendency to flip in certain special cases (low pressures and propagation of the wave almost perpendicular to the magnetic field.

Many problems of space and laboratory physics reduce to the need to investigate those characteristic plasma motions for which the length of the change in the parameters is small as compared with the length of the free path of the charged particles, but large as compared with their Larmor radius. Chew, Goldberger, and Low [1] have shown that in this case the behavior of the plasma can be described by a system of magnetohydrodynamic equations with anisotropic pressures. /3

Reference [2] used these equations to make a detailed analysis of the propagation of low amplitude waves in a collisionless plasma, and reference [3] investigated ordinary waves in plasma; an example of nonlinear waves, assuming that the longitudinal and transverse pressures ($p_{\parallel}$ and $p_{\perp}$) were small compared with $B^2/4\pi$, where $\vec{B}$ is the magnetic field induction vector. This paper is devoted to the investigation of ordinary waves in plasma within the Chew, Goldberger, and Low approximation, and without limitations as to the hydrodynamic parameters. The shapes of the integral curves for slow and fast magneto-sonic waves are obtained. It is proven that these waves have a tendency to flip in certain limit cases. The analysis for fast waves coincides,

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* Numbers in the margin indicate pagination in the foreign text.

and for slow waves does not, with the results in reference [3] in the limit case for low pressures.

1. Equations for Ordinary Waves /4

The set of magnetohydrodynamic equations in the Chew, Goldberger, and Low approximation is in the form (see [4], for example)

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \vec{v} = 0 \,; \quad \rho \frac{d\vec{v}}{dt} = -\operatorname{div} \overleftrightarrow{P} + \frac{1}{4\pi} \operatorname{rot} \vec{B} \times \vec{B} \,;$$

$$\frac{\partial \vec{B}}{\partial t} = \operatorname{rot} [\vec{v} \times \vec{B}] \,; \quad \operatorname{div} \vec{B} = 0 \,; \tag{1.1}$$

$$\frac{d}{dt}\left(\frac{p_{\parallel} B^2}{\rho^3}\right) = 0 \,; \quad \frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = 0$$

$$\operatorname{div} \overleftrightarrow{P} = \nabla p_{\perp} + (p_{\parallel} - p_{\perp})(\vec{b} \cdot \nabla)\vec{b} + \vec{b} \operatorname{div}(p_{\parallel} - p_{\perp})\vec{b} .$$

Here, ρ is the density,

$\vec{v}$ is the mean velocity,

$p_{\parallel}$ and $p_{\perp}$ are the longitudinal and transverse plasma pressures,

$\overleftrightarrow{P}$ is the stress tensor,

$\vec{B}$ is the magnetic field induction vector,

$\vec{b} = \vec{B}/B$ is the unit vector along the magnetic field.

We will consider those plasma motions for which all the parameters depend solely on one coordinate, x, and on time, t. Eq. (1.1) now can be rewritten in the form

$$\rho\left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}\right) = -(1-l)\frac{\partial p_{\perp}}{\partial x} - l \frac{\partial p_{\parallel}}{\partial x} + \frac{B_y}{B^2}\left[2l(p_{\parallel} - p_{\perp}) - \frac{B^2}{4\pi}\right]\frac{\partial B_y}{\partial x} + \frac{B_z}{B^2}\left[2l(p_{\parallel} - p_{\perp}) - \frac{B^2}{4\pi}\right]\frac{\partial B_z}{\partial x} \,;$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}\right)=\frac{B_xB_y}{B^2}\left(\frac{\partial p_\perp}{\partial x}-\frac{\partial p_\parallel}{\partial x}\right)+\frac{B_x}{B^2}\left[p_\perp-p_\parallel-2(p_\perp-p_\parallel)\frac{B_y^2}{B^2}+\right.$$
$$\left.+\frac{B^2}{4\pi}\right]\frac{\partial B_y}{\partial x}-2(p_\perp-p_\parallel)\frac{B_xB_yB_z}{B^4}\frac{\partial B_z}{\partial x};$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}\right)=\frac{B_xB_z}{B^2}\left(\frac{\partial p_\perp}{\partial x}-\frac{\partial p_\parallel}{\partial x}\right)+\frac{B_x}{B^2}\left[p_\perp-p_\parallel-2(p_\perp-p_\parallel)\frac{B_z^2}{B^2}+\right.$$
$$\left.+\frac{B^2}{4\pi}\right]\frac{\partial B_z}{\partial x}-2(p_\perp-p_\parallel)\frac{B_xB_yB_z}{B^4}\frac{\partial B_y}{\partial x} \qquad (1.2)$$

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+\rho\frac{\partial u}{\partial x}=0;\quad \frac{\partial B_y}{\partial t}+u\frac{\partial B_y}{\partial x}-B_x\frac{\partial v}{\partial x}+B_y\frac{\partial u}{\partial x}=0;$$
$$\frac{\partial B_z}{\partial t}+u\frac{\partial B_z}{\partial x}-B_x\frac{\partial w}{\partial x}+B_z\frac{\partial u}{\partial x}=0$$

$$\frac{\partial p_\perp}{\partial t}+u\frac{\partial p_\perp}{\partial x}-\frac{p_\perp}{\rho}\left(\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}\right)-\frac{p_\perp B_y}{B^2}\left(\frac{\partial B_y}{\partial t}+u\frac{\partial B_y}{\partial x}\right)-\frac{p_\perp B_z}{B^2}\left(\frac{\partial B_z}{\partial t}+u\frac{\partial B_z}{\partial x}\right)=0,$$

$$\frac{\partial p_\parallel}{\partial t}+u\frac{\partial p_\parallel}{\partial x}-\frac{3p_\parallel}{\rho}\left(\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}\right)+\frac{2p_\parallel B_y}{B^2}\left(\frac{\partial B_y}{\partial t}+u\frac{\partial B_y}{\partial x}\right)+\frac{2p_\parallel B_z}{B^2}\left(\frac{\partial B_z}{\partial t}+u\frac{\partial B_z}{\partial x}\right)=0;$$
$$\frac{\partial B_x}{\partial t}+u\frac{\partial B_x}{\partial x}=0;\quad \frac{\partial B_x}{\partial x}=0\quad\left(\ell=\frac{B_x^2}{B^2}\right).$$

We will seek those solutions of this set for which all magnitudes depend solely on the combination $\varphi(x, t)$ of independent variables x and t (ordinary waves, or Riemann waves). Moreover, what follows from the last two equations of Eq. (1.2) is that B_x = constant. Introducing the velocity of wave phase motion relative to the gas through the formula

$$\alpha = \lambda - u, \tag{1.3}$$

where

λ is the phase velocity of an ordinary wave, we obtain from Eq. (1.2) a homogeneous linear set of equations for the derivatives of the unknown functions

$$-\rho\alpha u' + (1-\ell)p_\perp' + \ell p_\parallel' + \frac{B_y}{B^2}\left[\frac{B^2}{4\pi} - 2(p_\parallel - p_\perp)\ell\right]B_y' - \frac{B_z}{B^2}\left[2(p_\parallel - p_\perp)\ell - \frac{B^2}{4\pi}\right]B_z' = 0\ ;$$

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$$-\rho\alpha v' + \frac{B_x B_y}{B^2}(p_\parallel' - p_\perp') + \frac{B_x}{B^2}\left[p_\parallel - p_\perp - 2(p_\parallel - p_\perp)\frac{B_y^2}{B^2} - \frac{B^2}{4\pi}\right]B_y' - 2(p_\parallel - p_\perp)\frac{B_x B_y B_z}{B^4}B_z' = 0\ ;$$

$$-\rho\alpha w' + \frac{B_x B_z}{B^2}(p_\parallel' - p_\perp') + \frac{B_x}{B^2}\left[p_\parallel - p_\perp - 2(p_\parallel - p_\perp)\frac{B_z^2}{B^2} - \frac{B^2}{4\pi}\right]B_z' - 2(p_\parallel - p_\perp)\frac{B_x B_y B_z}{B^4}B_y' = 0\ ; \tag{1.4}$$

$$-\alpha\rho' + \rho u' = 0\ ; \quad -\alpha B_y' + B_y u' - B_x v' = 0\ ;$$

$$-\alpha B_z' + B_z u' - B_x w' = 0\ ;$$

$$-\alpha p_\perp' + \frac{p_\perp}{\rho}\alpha\rho' + \frac{p_\perp B_y}{B^2}\alpha B_y' + \frac{p_\perp B_z}{B^2}\alpha B_z' = 0\ ;$$

$$-\alpha p_\parallel' + \frac{3p_\parallel}{\rho}\alpha\rho' - \frac{2p_\parallel B_y}{B^2}\alpha B_y' - \frac{2p_\parallel B_z}{B^2}\alpha B_z' = 0\ ;$$

Here the prime designates differentiation in terms of the function $\omega(x, t)$. The determinant of this set should equal zero in order for there to be nontrivial solutions. Calculation of the determinant for Eq. (1.4) reduces to a characteristic equation, which has the form

$$\mathcal{D}(a) \equiv a^2 \left[F(a^2) - \ell(1-\ell) p_\perp^2 \right] \left[a^2 - \frac{1}{\rho}\left(\frac{B^2}{4\pi} + p_\perp - p_\parallel \right)\ell \right] = 0. \tag{1.5}$$

Here

$$F(a^2) = \left[\rho a^2 - \ell\left(\frac{B^2}{4\pi} + p_\perp - p_\parallel \right) - 2(1-\ell)\left(p_\perp + \frac{B^2}{8\pi}\right)\right]\left(\rho a^2 - 3\ell p_\parallel\right). \tag{1.6}$$

Naturally enough, the characteristic Eq. (1.5) of ordinary waves coincides /7
with the characteristic equation of small amplitude waves in reference [2]. The results obtained in [2] are then used to investigate ordinary waves. The solution of characteristic Eq. (1.5) results in the following values of a for which the set of equations of Eq. (1.4) has nontrivial solutions

$$a_{1,2} = 0 \; ; \qquad a_{3,4} = \pm a_A \; ; \quad a_{5-8} = \pm a_{+,-} \; ;$$

where

$$a_A^2 = \frac{1}{\rho}\left(\frac{B^2}{4\pi} + p_\perp - p_\parallel \right)\ell \; ; \tag{1.7}$$

$$a_{+,-}^2 = \frac{p_\perp}{\rho} + \frac{B^2}{8\pi\rho} + \frac{1}{2\rho}(2p_\parallel - p_\perp)\ell \pm \left\{ \left[\frac{p_\perp}{\rho} + \frac{B^2}{8\pi\rho} + \frac{1}{2\rho}(2p_\parallel - p_\perp)\ell \right]^2 + \left[\left(\frac{p_\perp}{\rho}\right)^2 \ell(1-\ell) - \frac{3p_\parallel p_\perp}{\rho^2}\ell(2-\ell) + 3\left(\frac{p_\parallel}{\rho}\ell\right)^2 - \frac{3p_\parallel}{\rho}\ell\frac{B^2}{4\pi\rho} \right] \right\}^{1/2}. \tag{1.8}$$

The zero root, as well as the roots of a_A and $a_{+,\,-}$, define the solutions, which are an analogue of an entropy wave, of an Alfén wave, as well as of fast and slow magneto-sonic waves in magnetohydrodynamics with isotropic pressure [5]. The analogue of Alfén waves was investigated in [3], and the analogue of entropy waves can be investigated readily by using the set of equations of Eq. (1.4). In what follows, only fast and slow magneto-sonic waves will be investigated in detail because there is a physical meaning to entropy waves /8
only after the introduction of dissipative mechanisms in the set of Chew, Goldberger, and Low equations, and the introduction of the concept of entropy in a collisionless plasma with anisotropic pressure.

Reference [2] obtained the inequalities

$$\varrho a_+^2 \geq \max\{3\ell p_\parallel;\ (1-\ell)p_\perp + 3\ell p_\parallel + p_M - 4\ell p_\parallel\}, \qquad (1.9)$$

$$\varrho a_-^2 \leq \min\{3\ell p_\parallel;\ (1-\ell)p_\perp + 3\ell p_\parallel + p_M - 4\ell p_\parallel\};$$

where

$$p_M = p_\perp + \frac{B^2}{4\pi}.$$

Moreover, [2] showed that the root of the slow magneto-sonic wave is real only when the inequality is satisfied (what follows from the above-described inequalities is that the a_+ root is always real)

$$p_m \equiv \frac{p_\perp^2}{6p_\perp + 3\frac{B^2}{4\pi}} \leq p_\parallel\ ; \qquad (1.10)$$

and that the regions of change in the longitudinal pressure, in which the root of the characteristic Eq. (1.5) is real, can be broken down into three parts

$$p_m \leq p_\parallel < \frac{1}{4}p_M;\ (a_- \leq a_A \leq a_+) \qquad (1.11)$$

$$\frac{1}{4}p_M < p_\parallel < \frac{1}{4}(p_M + 3p_m);\ (a_- \leq a_A < a_+); \qquad (1.12)$$ /9

$$\frac{1}{4}(p_M + 3p_m) < p_\parallel \leq p_M \equiv p_\perp + \frac{B^2}{4\pi}; \quad (a_A \leq a_- < a_+). \tag{1.13}$$

It should be noted that when $p_\parallel > p_M$, a so-called "hose" instability develops in the plasma (the Alfén wave in this case has a purely imaginary root).

2. Investigation of Fast and Slow Magneto-sonic Waves

Let us put

$$a = a_{+,\,-}$$

in the set of equations of Eq. (1.4). Now, if we select our system of coordinates such that at some arbitrary point the condition $w = B_z = 0$, is satisfied, it can be said that what follows from Eq. (1.4) is

$$w' = B_z' = 0.$$

And, just as in [5], it can be put everywhere that $w = B_z = 0$. Moreover, taking the plasma density ρ for the function $\varphi(x, t)$, and solving the set of equations in terms of the derivatives, we obtain from Eq. (1.4) the following set of equations describing the change of parameters in fast and slow magneto-sonic waves (at the same time, it is easy to show that the fifth equation in Eq. (1.4) is linearly dependent, and it can be discarded). /10

$$\frac{dB_y^2}{d\rho} = \frac{2B^2}{\rho} \cdot \frac{A_{+,-}}{C}; \quad \frac{du}{d\rho} = \frac{a_{+,-}}{\rho}; \tag{2.1}$$

$$\frac{d}{d\rho}\left(\frac{p_\perp}{\rho B}\right) = 0; \quad \frac{d}{d\rho}\left(\frac{p_\parallel B^2}{\rho^3}\right) = 0;$$

$$\frac{dv}{d\rho} = \frac{B_x}{\rho^2 a_{+,-} B_y}\left\{(1-\ell)(3p_\parallel - p_\perp) + \left[p_\parallel - p_M - (4p_\parallel - p_\perp)(1-\ell)\right]\frac{A_{+,-}}{C}\right\}.$$

Here

$$A_{+,-} = \rho a^2_{+,-} - (1-\ell)p_\perp - 3\ell p_\parallel \,; \quad C = p_M - \ell(4p_\parallel - p_\perp)\,;$$

$$\ell = \frac{B_x^2}{B^2} \,; \quad B^2 = B_x^2 + B_y^2 \qquad (2.2)$$

The magnitude $A_{+,-}$ is readily converted into

$$A_{+,-} = \frac{1}{2}\left[\frac{B^2}{4\pi} - \ell(4p_\parallel - p_\perp) \pm \sqrt{\left[\frac{B^2}{4\pi} - \ell(4p_\parallel - p_\perp)\right]^2 + 4(1-\ell)p_\perp C}\,\right]. \qquad (2.3)$$

As will be seen from Eq. (2.1), in ordinary waves

$$\frac{p_\parallel B^2}{\rho^3} = const \equiv C_1 \,; \quad \frac{p_\perp}{\rho B} = const \equiv C_2 . \qquad (2.4)$$

Moreover, when $B_y^2 \to 0$ $(b \to 1)$ /11

$$A_{+,-} = \frac{1}{2}\left[\frac{B_x^2}{4\pi} - g(\rho) \pm \left|\frac{B_x^2}{4\pi} - g(\rho)\right|\right]; \qquad (2.5)$$

$$C = p_M^* - g(\rho),$$

where

$$g(\rho) \equiv 4p_\parallel^* - p_\perp^* = C_1^* \rho^3 - C_2^* \rho \,; \quad (B_x = const)\,; \qquad (2.6)$$

The superscript (*) signifies that the functions are taken when $B_y^2 = 0$, and C_1*, C_2* are constants. The function $g(\rho)$ is in the form shown in Figure 1. It is obvious that $p_\parallel \to 0$ when $\rho \to 0$, and $p_\perp \to 0$ in this case if B_y is limited. But when $B_y \gg B_x$ $(b \to 0)$ we have, for a fast wave,

$$\frac{dB_y^2}{d\rho} = \frac{2B_y^2}{\rho} \quad \text{or} \quad \frac{B_y}{\rho} = const, \qquad (2.7)$$

and for a fast wave when $B_y \gg B_x$

$$p_\perp + \frac{B_y^2}{8\pi} = const. \qquad (2.8)$$

Now let us move on to the construction of the integral curve field.

(a) Fast magneto-sonic wave /12

It will be seen from Eq. (2.3) for A_+, and from Eq. (2.2) for C that the sign A_+ everywhere coincides with the sign of C. Specifically, it is easy to show that in the ranges defined by the inequalities of Eqs. (1.11) and (1.12) we have $A_+ > 0$ and $C > 0$, for any values of the parameter $b = B_x^2/B^2 \leq 1$, and in the range defined by the inequalities of Eq. (1.13) the value of C can change signs (at the same time we have $A_+ = 0$ at the point where $C = 0$).

Thus, it can be concluded that just as in the case of magnetohydrodynamics with isotropic pressure [5], the magnetic field in the compression wave increases, and decreases in the rarefaction wave.

When $B_y^2 \to 0$ when ρ are small, and as will be seen from Figure 1, we have $g(\rho) < B_x^2/4\pi$, and from Eqs. (2.5) and (2.6) we obtain

$$A_+ = \frac{B_x^2}{4\pi} + p_\perp^* - 4p_\parallel^* \equiv \frac{B_x^2}{4\pi} - g(\rho) > 0 ;$$

when $\rho \geq \rho^*$, from Eqs. (2.5) and (2.6) we have $A_+ = 0$, where ρ^* can be established through the equation

$$g(\rho^*) = \frac{B_x^2}{4\pi}$$

and when $\rho \to 0$ we have $A_+ \to B^2/4\pi$, $C \to B^2/4\pi$. Therefore $dB_y^2/d\rho \to \infty$ $(B \neq 0)$.

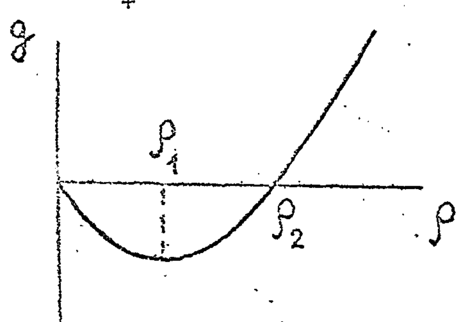

Figure 1.

It is also easy to show that the derivative $dB_y^2/d\rho$ when $B_y^2 \to 0$ is a decreasing function of ρ. Considering everything reviewed in the foregoing, as well as Eq. (2.7), the integral curve field in the plane (ρ, B_y^2) for a fast magneto-sonic wave can be presented in the form shown in Figure 2. In this figure, curve 1 is found through the equation /13

$$\rho_{\parallel} = \rho_{\perp} + \frac{B^2}{4\pi} \equiv \rho_M .$$

To the right of this curve, the fast magneto-sonic wave enters the region of "hose" instability.

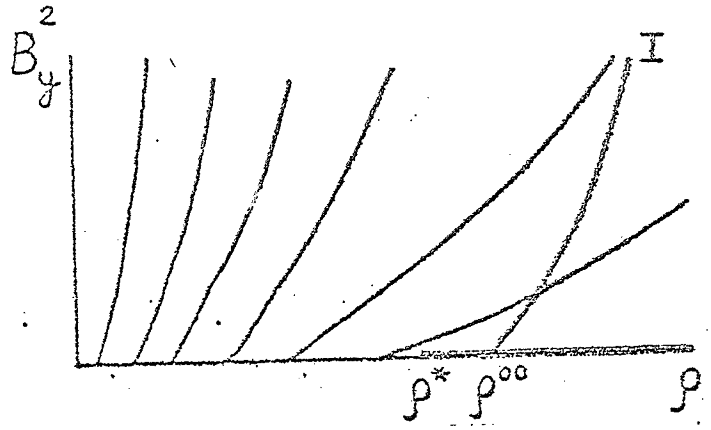

Figure 2.

(b) Slow magneto-sonic wave

As will be seen from Eqs. (1.9) and (2.2), $A_- \leq 0$ in a slow magneto-sonic wave. We therefore have $dB_y^2/d\rho \geq 0$ in the region where $C > 0$. We have $A_- < 0$ at the point where $C = 0$, and the derivative tends to infinity; that is, at this point there is no change in density with increase in the magnetic field. We have $dB_y^2/d\rho > 0$ in a slow magnetic wave in the region where $C < 0$. When $B_y^2 \to 0$, we have $dB_y^2/d\rho = 0$ when $\rho \leq \rho^*$, and $dB_y^2/d\rho < 0$ when $\rho^* < \rho < \rho^{**}$, where ρ^{**} is found through the equation

$$\rho_M^* - 4\rho_{\parallel}^* + \rho_{\perp}^* = 0.$$

We have $dB_y^2/d\rho \to \infty$ at the point $\rho = \rho^{**}$ when $B_y^2 \to 0$, and when $\rho > \rho^{**}$ we have $dB_y^2/d\rho > 0$. But when $B_y^2 \gg B_x^2$ ($b \to 0$) we have $dB_y^2/d\rho < 0$. From whence /14 it follows that the integral curve passing through the point ($\rho > \rho^{**}$, $B_y^2 = 0$) changes the sign of the derivative. At the same time, using Eq. (2.4), it is easy to show that the sign of the derivative will change only once.

When $\rho \to 0$, we have $C \to B^2/4\pi$, $A_- \to -(1-b)p_\perp$, and as a result, $dB_y^2/d\rho \to -8\pi(1-b)C_2B$. Thus, when $\rho \to 0$, we have $dB_y^2/d\rho < 0$. This derivative is limited, and more so in terms of the absolute magnitude than by B^2. The integral curve field in the plane (ρ, B_y^2) has the shape shown in Figure 3 for a slow magneto-sonic wave. In this figure, the dashed curve is found through the equation

$$p_m \equiv \frac{p_\perp^2}{6p_\perp + 3B^2/4\pi} = p_{\parallel} .$$

To the left of this curve $p_{\parallel} < p_m$, and the slow magneto-sonic wave will become imaginary. It is obvious that the postion of curve 1, and of the dashed line, as shown in Figures 2 and 3, are significantly dependent on the constants C_1 and C_2 in Eq. (2.4).

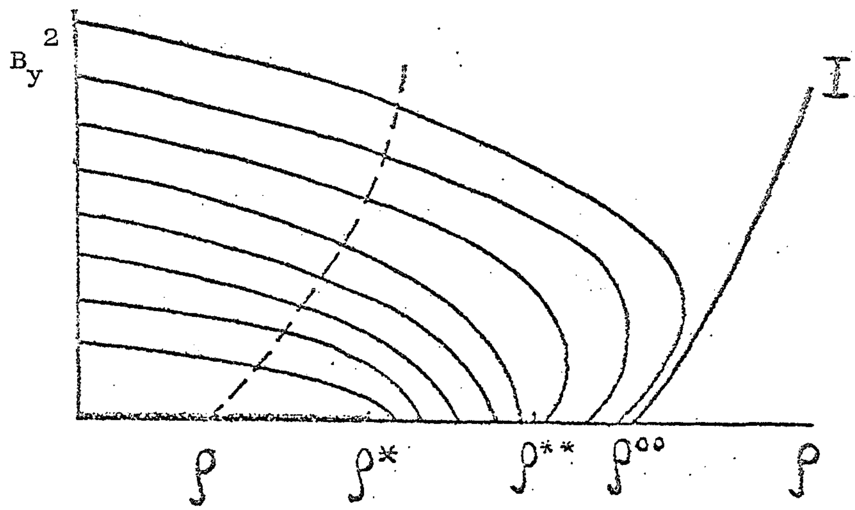

Figure 3.

3. Some Additional Results

Curve 1 in Figures 2 and 3, and the dashed curve in Figure 3, are found through the equations

$$p_{\parallel} = p_M \quad ; \quad p_{\parallel} = p_m , \tag{15}$$

which, with Eq. (2.4) taken into consideration, have the following forms, respectively,

$$\frac{C_1 \rho^3}{B_x^2 + B_y^2} = C_2 \rho \sqrt{B_x^2 + B_y^2} + \frac{B_x^2 + B_y^2}{4\pi} ; \tag{3.1}$$

$$\rho = -\frac{\sqrt{B_x^2 + B_y^2}}{16\pi C_2} + \sqrt{\frac{B_x^2 + B_y^2}{(16\pi C_2)^2} + \frac{C_2 (B_x^2 + B_y^2)^{3/2}}{6 C_1}} ; \tag{3.2}$$

As will be seen from Eq. (3.2), when $B_y^2 \to 0$, we have

$$\rho \to \rho^\circ = -\frac{B_x}{16\pi C_2} + \sqrt{\frac{B_x^2}{(16\pi C_2)^2} + \frac{C_2 B_x^3}{6 C_1}} ,$$

and, obviously, $\rho^\circ < \rho^*$ (because $p_m < 1/4\, p_M$). It is easy to show that for the Eq. (3.2) curve we have $d\rho/dB_y^2 > 0$ and $d\rho/dB_y^2 \to 0$ when $B_y \to \infty$, and that the Eq. (3.1) curve intersects the $B_y^2 = 0$ axis at the point $\rho^{\circ\circ}$ satisfying the inequality

$$\rho^{\circ\circ} > \rho^{**}.$$

Moreover, differentiating Eq. (3.1) with respect to ρ, it can be shown that at the point ($\rho^{\circ\circ}$, $B_y^2 = 0$) the magnetic field along the integral curve for a slow magneto-sonic wave increases more rapidly than does the magnetic field along the Eq. (3.1) curve (curve 1 in Figure 3 corresponds to the equation $p_\parallel = p_M$).

In the limit case $p_\perp \ll B^2/4\pi$ and $p_\parallel \ll B^2/4\pi$ equations, and the entire analysis for the fast magneto-sonic wave, coincide with reference [3] results. Specifically, for the fast magneto-sonic wave we have /16

$$\frac{d\lambda}{d\rho} = \frac{d}{d\rho}(u + a_+) > 0 ,$$

where λ is found through Eq. (1.3); that is, wave flip occurs.

We have the following for the slow magneto-sonic wave in this case

$$\frac{dB_y^2}{d\rho} = -(1-l)\frac{8\pi p_\perp}{\rho} ; \quad \frac{du}{d\rho} = \frac{a_-}{\rho} ; \quad \frac{dv}{d\rho} = \frac{(1-l)B_y}{\rho B_x} a_- ;$$

$$\frac{dp_\perp}{d\rho} = \frac{p_\perp}{\rho} ; \quad \frac{dp_\parallel}{d\rho} = \frac{3p_\parallel}{\rho} ; \quad a_-^2 = \frac{3p_\parallel}{\rho} l ,$$

and this does not coincide with the equations and the analysis obtained in reference [3]. Specifically, the anomaly case is missing in this limit case and everywhere $d\lambda/d\rho > 0$; that is, slow magneto-sonic wave flip occurs. In the general case we have

$$\frac{d\lambda}{d\rho} = \frac{du}{d\rho} + \frac{da_{+,-}}{d\rho} = \frac{1}{\rho}\,\frac{d\rho a_{+,-}}{d\rho}.$$

Differentiating the characteristic equation for fast and slow magneto-sonic waves, and after cumbersome transformations, we obtain

$$a_1 \frac{d\lambda}{d\rho} = a_2 \frac{A_{+,-}}{C} + a_3 , \tag{3.3}$$

$$a_1 = 2\rho^2 a_{+,-}\left[2\rho a_{+,-}^2 - p_\perp - p_M - l(2p_\parallel - p_\perp)\right]; \tag{3.4}$$ /17

$$a_2 = \rho a_{+,-}^2\left[2p_M - l(8p_\parallel - p_\perp)\right] + 3l(6-5l)p_\parallel p_\perp + 2l^2 p_\perp^2 - 24l^2 p_\parallel^2 + 6lp_\parallel \frac{B^2}{4\pi}; \tag{3.5}$$

$$a_3 = \rho a_{+,-}^2\left[3p_\perp + p_M + 2l(4p_\parallel - p_\perp)\right] - 18l(2-l)p_\parallel p_\perp + 4l(1-l)p_\perp^2 + 24l^2 p_\parallel^2 - 15lp_\parallel \frac{B^2}{4\pi} \tag{3.6}$$

If the Riemann wave is propagated almost perpendicular to the magnetic field ($b \to 0$), we have, for the fast wave

$$\frac{d\lambda}{d\rho} \to \frac{3}{2\rho}\sqrt{\frac{1}{\rho}\left(2p_\perp + \frac{B^2}{4\pi}\right)} > 0 .$$

In this case for the slow magneto-sonic wave we have

$$a_-^2 \to \frac{3l}{\rho}(p_\parallel - p_M); \quad A_- \to -p_\perp; \quad C \to p_M . \tag{3.7}$$

After uncomplicated transformations, using Eqs. (3.3) through (3.7), we obtain

$$\frac{d\lambda}{d\xi} = \frac{l}{2\xi^2 a_-(p_\perp + p_M)}\Big[4(p_\parallel - p_m)\Big(6p_\perp + 3\frac{B^2}{4\pi}\Big) +$$

$$+ \frac{6p_\parallel p_\perp}{p_M}(2p_\perp + p_M) + 6p_\parallel p_\perp + 3p_m(p_\perp + p_M)\Big] > 0 ,$$

that is, when a slow magneto-sonic wave is propagated almost perpendicular to /18
the magnetic field in the region in which this wave ($p_\parallel \geq p_m$) exists, it has a tendency to flip.

Designations Used

$\vec{v}$	plasma velocity	/20
ρ	density	
$p_\parallel$, $p_\perp$	longitudinal and transverse pressures	
$\vec{B}$	magnetic induction vector	
$\vec{b}$	unit vector along the magnetic induction vector	
λ	wave phase velocity	
$A_{+,-}$	velocity of fast and slow magneto-sonic waves, respectively	
a_A	Alfén wave velocity	

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Translated for the National Aeronautics and Space Administration under contract No. NASw-2038 by Translation Consultants, Ltd., 944 South Wakefield Street, Arlington, Virginia 22204